For the potential well shown in the figure, consider two energy eigenstates, one with energy corresponding to the blue line ($E_1 < V_0$) and one corresponding to the orange line ($E_2 > V_0$).

1. Sketch the wavefunction
2. Identify the classically allowed and forbidden regions
3. Discuss the important qualitative features.

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Some things to pay attention to:

1. **Boundary Conditions:**
   a) Both energy eigenstates should go to zero where the potential is infinite, at $x = 0$ and $x = L$.
   b) Both the wavefunction and the derivative of the wavefunction should be continuous at $x = L/2$.
2. **Oscillatory vs. Exponential Shape:**
   a) In regions where $E > V$, then you have oscillatory solutions.
   $$\frac{\partial^2\phi(x)}{\partial x^2} = \underbrace{-\frac{\sqrt{2m(E-V)}}{\hbar}}_{-k^2}\phi(x)$$
   $$\phi(x) = A\sin kx + B\cos kx$$
   b) In regions where $E < V$, then you have decaying exponetial solutions.
   $$\frac{\partial^2\phi(x)}{\partial x^2} = \underbrace{-\frac{\sqrt{2m(E-V)}}{\hbar}}_{q^2}\phi(x)$$
   $$\phi(x) = Ae^{qx} + Be^{-kx}$$
3. **Number of Wiggles:** For oscillatory solutions, the wavenumber is related to the different between the total energy and the potential energy:
   $$k = \frac{\sqrt{2m(E-V)}}{\hbar}$$
   The larger the difference, the larger the wavenumber. Large wavenumber means a large number of waves per meter (in other words, a lot of wiggles).

4. **Amplitude:** The norm squared of the wavefunction corresponds to the probability density. So, in regions where the particle spends a lot of time, the amplitude should be bigger. The particle spends more time in regions where it's kinetic energy is smaller. So, if $E - V$ is smaller, the amplitude should be bigger.