The Gaussian

$$\mathcal{P}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \tag{1}$$

is normalized so that the area under the curve is equal to one. If this Gaussian represents the probability density for a free quantum mechanical particle, what is a possible wavefunction?

Solution The probability density is the square of the norm of the wavefunction. So, take the square root to find the wavefunction.

The wavefunction is generally complex, and a complex phase $e^{i\gamma}$ will disappear when taking the norm square to get the probability density, so I need to remember to account for it when I take the square root:

$$\begin{aligned}
\psi(x) &= \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \right)^{1/2} \\
&= e^{i\gamma} \left(\frac{1}{2\pi\sigma^2} \right)^{1/4} e^{-\frac{(x-x_0)^2}{4\sigma^2}}
\end{aligned}$$

This function is still a Gaussian, only the width has changed! (It's increased by a factor of $\sqrt{2}$.)