In the **position** representation (basis), the position operator is a multiplication by x:

$$\hat{x}\,|x_0\rangle = x_0\,|x_0\rangle$$

$$\hat{x}\,\langle x|x_0\rangle = x_0\,\langle x|x_0\rangle$$

$$\hat{x}\delta(x - x_0) = x_0\delta(x - x_0)$$

The momentum operator is a differential operator:

$$\hat{p}\,|p_0\rangle = p_0\,|p_0\rangle$$

$$\hat{p}\,\langle x|p_0\rangle = p_0\,\langle x|p_0\rangle$$

$$\hat{p}\frac{e^{ip_0x/\hbar}}{\sqrt{2\pi\hbar}} = \left(-i\hbar\frac{\partial}{\partial x}\right)\frac{e^{ip_0x/\hbar}}{\sqrt{2\pi\hbar}}$$

$$= -i\hbar\left(i\frac{p_0}{\hbar}\frac{e^{ip_0x/\hbar}}{\sqrt{2\pi\hbar}}\right)$$

$$= p_0\frac{e^{ip_0x/\hbar}}{\sqrt{2\pi\hbar}}$$

In the **momentum** representation (basis):

$$|p_0\rangle \doteq \langle p|p_0\rangle = \delta(p - p_0)$$

$$|x_0\rangle \doteq \langle p|x_0\rangle = e^{-ipx_0/\hbar}$$

What do the position and momentum operators look like in the momentum representation?

Solution   The calculations proceed in exactly the same way:

$$\hat{p}\,|p_0\rangle = p_0\,|p_0\rangle$$

$$\hat{p}\,\langle p|x_0\rangle = p_0\,\langle p|p_0\rangle$$

$$\hat{p}\delta(p - p_0) = p_0\delta(p - p_0)$$

$$\hat{x}\ket{x_0} = x_0\ket{x_0}$$

$$\hat{x}\braket{p|x_0} = x_0\braket{p|x_0}$$

$$\hat{x}\frac{e^{-ipx_0/\hbar}}{\sqrt{2\pi\hbar}} = \left(i\hbar\frac{\partial}{\partial p}\right)\frac{e^{-ipx_0/\hbar}}{\sqrt{2\pi\hbar}}$$

$$= i\hbar\left(-i\frac{x_0}{\hbar}\frac{e^{-ipx_0/\hbar}}{\sqrt{2\pi\hbar}}\right)$$

$$= x_0\frac{e^{ipx_0/\hbar}}{\sqrt{2\pi\hbar}}$$

Notice that the the momentum operator in the position representation and the position operator in the momentum representation are each derivatives but are different by a minus sign. This minus sign comes from the fact that the momentum eigenstate in the position representation and the position eigenstate in the momentum representation are complex conjugates of each other:

$$\braket{x|p} = \braket{p|x}^*$$