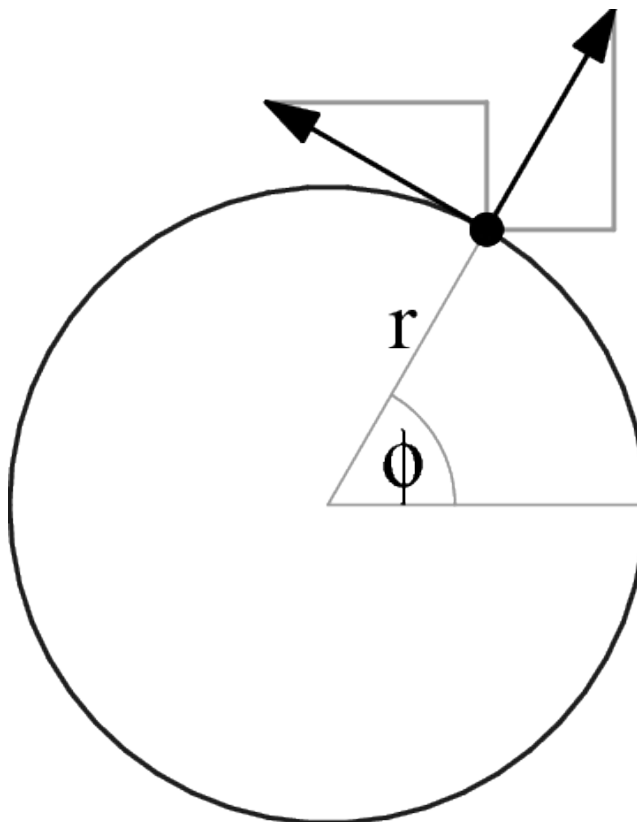




In rectangular coordinates, the natural unit vectors are $\{\hat{x}, \hat{y}\}$, which point in the direction of increasing x and y , respectively. Similarly, in polar coordinates the natural unit vectors are $\hat{r}$, which points in the direction of increasing r , and $\hat{\phi}$, which points in the direction of increasing ϕ .

The *unit tangent vector* to a parametric curve is the unit vector tangent to the curve which points in the direction of increasing parameter. The *principal unit normal vector* to a parametric curve is the unit vector perpendicular to the curve “in the direction of bending”, which is the direction of the *derivative* of the unit tangent vector.

1. Consider the parametric curve $\vec{r} = 3 \cos \phi \hat{x} + 3 \sin \phi \hat{y}$ with $\phi \in [0, 2\pi]$. Calculate the unit tangent vector $\hat{T}$ and the principal unit normal vector $\hat{N}$ for this curve in terms of $\hat{x}$ and $\hat{y}$.

Solution $\hat{T} = -\sin \phi \hat{x} + \cos \phi \hat{y}$
 $\hat{N} = -(\cos \phi \hat{x} + \sin \phi \hat{y})$

2. Consider a circle of radius 3 centered at the origin. Determine the unit tangent vector $\hat{T}$ and the principal unit normal vector $\hat{N}$ for this curve in terms of $\hat{r}$ and $\hat{\phi}$.

Solution $\hat{T} = \hat{\phi}$
 $\hat{N} = -\hat{r}$

3. Compare your answers.