Consider the vector field given by *(μ_0 and I are constants)*: $\vec{\boldsymbol{B}} = \frac{\mu_0 I}{2\pi}\left(\frac{-y\,\hat{\boldsymbol{x}}+x\,\hat{\boldsymbol{y}}}{x^2+y^2}\right) = \frac{\mu_0 I}{2\pi}\,\frac{\hat{\boldsymbol{\phi}}}{s}$

$\vec{\boldsymbol{B}}$ is the magnetic field around a wire along the z-axis carrying a constant current I in the z-direction.

Ready:

- Determine $\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}}$ on any radial line of the form $y = mx$, where m is a constant.
- Determine $\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}}$ on any circle of the form $x^2+y^2=a^2$, where a is a constant.
  You may wish to express the equations for these curves in polar coordinates.

Solution On a radial line, $d\vec{\boldsymbol{r}} = dr\,\hat{\boldsymbol{r}}$, so $\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}} = 0$. On a circle centered at the origin, $d\vec{\boldsymbol{r}} = r\,d\phi\,\hat{\boldsymbol{\phi}}$, so $\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}} = \frac{\mu_0 I}{2\pi}\,d\phi$

Go: For each of the following curves C_i, evaluate the line integral $\int_{C_i}\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}}$.

- C_1, the *top* half of the circle $s=5$, traversed in a *counterclockwise* direction.

Solution $\frac{\mu_0 I}{2}$

- C_2, the *top* half of the circle $s=2$, traversed in a *counterclockwise* direction.

Solution $\frac{\mu_0 I}{2}$

- C_3, the *top* half of the circle $s=2$, traversed in a *clockwise* direction.

Solution $-\frac{\mu_0 I}{2}$

- C_4, the *bottom* half of the circle $s=2$, traversed in a *clockwise* direction.

Solution $-\frac{\mu_0 I}{2}$

- C_5, the radial line from $(2,0)$ to $(5,0)$.

Solution 0

- C_6, the radial line from $(-5,0)$ to $(-2,0)$.

Solution 0

FOOD FOR THOUGHT

- Construct ***closed*** curves C_7 and C_8 such that this integral $\int_{C_i}\vec{\boldsymbol{B}}\cdot d\vec{\boldsymbol{r}}$ is nonzero over C_7 and zero over C_8.
  *It is enough to draw your curves; you do **not** need to parameterize them.*

- Ampère's Law says that, for any closed curve C, this integral is (μ_0 times) the current flowing ***through*** C (in the z direction). Can you use this fact to explain your results to part (a)?
- Is $\vec{\boldsymbol{B}}$ conservative?

Solution Many answers are possible. The integral will be nonzero if and only if the closed path goes around the origin. This vector field represents the magnetic field of a current-carrying wire located along the z axis.

As for whether $\vec{\boldsymbol{B}}$ is conservative, that depends on the allowed domain. It is clearly *not* conservative in the usual sense, since there are closed paths that yield nonzero integrals. But it *does* satisfy the properties of a conservative vector field so long as the domain is restricted to prevent closed paths that go around the origin, such as by removing the positive x axis.