The momentum operator is $\hat{p} = -i\hbar\frac{\partial}{\partial x}$.

1. Write down the momentum eigenvalue equation.

Solution

$$\hat{p}\ |p\rangle = p\ |p\rangle$$

$$\hat{p}\,\psi_p(x) = p\,\psi_p(x)$$

$$-i\hbar\frac{\partial}{\partial x}\psi_p(x) = p\,\psi_p(x)$$

2. Solve for the momentum eigenstates.

Solution

$$-i\hbar\frac{\partial}{\partial x}\psi_p(x) = p\,\psi_p(x)$$

$$\frac{\partial}{\partial x}\psi_p(x) = \frac{ip}{\hbar}\,\psi_p(x)$$

$$\rightarrow \quad \psi_p(x) = Ae^{ipx/\hbar}$$