Suppose you have a definite function $f(x)$ in mind and you already know its Fourier transform, i.e. you know how to do the integral

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, f(x)\, dx \tag{1}$$

Find the Fourier transform of the shifted function $f(x - x_0)$.

Solution Make the substitution $y = x - x_0$, $dy = dx$:

$$\begin{aligned} \tilde{f}_{x_0}(k) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, f(x - x_0)\, dx && (2) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ik(y+x_0)}\, f(y)\, dy && (3) \\ &= e^{-ikx_0} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iky}\, f(y)\, dy \right) && (4) \\ &= e^{-ikx_0}\, \tilde{f}(k) && (5) \end{aligned}$$

Notice that the shift in the original function just resulted in an extra phase factor multiplying the Fourier transform.