Estimate the average rate of electrical energy production (J/s) that is possible using the gravitational potential energy of rain water collected on the roof of a typical family home in Corvallis (averaged over an entire year). Assume that you can capture the gravitational potential energy of the water as it drops from gutter height to ground level.

Solution Here is one way to answer the question.

The "leading" variables (those which have the most affect) are roof area, roof height, and rainfall. I'll ignore everything else.

Less impactful variables are diameter of downspout, the style of the water turbine, etc.

Google says 50 inch rainfall per year = 1.25 m/year (it's helpful to memorize 1 in $\approx$ 2.5 cm).

Roof area is about 1500-2500 ft^2. My house has 1500 ft^2 not counting the garage. 1 m $\approx$ 3 ft, so 1 m^2 $\approx$ 9 ft^2. So let's call our roof area 200 m^2. There is a large variation in roof area, therefore we can move forward with minimal precision. This means I will round numbers at each step in the calculation (which makes my calculation quick, easy to adapt, and easy to follow).

I have a one-story house, so my gutters are about 2 m off the ground.

The volume of water that lands on the roof in one year is thus about 200 m/year$^2\times$1.25 m, which gives us about 250 m^3/year. The density of water is 1 g/cm^3 or 10^3 kg/m^3, so the mass rate of of rain on the roof is 2.5$\times 10^5$ kg/year.

The potential energy is $mg\Delta h$, so we can find the rate of energy as

$$\begin{aligned}
\text{energy rate} &= (2.5 \times 10^5 \text{ kg/year})(10 \text{ m/s}^2)(2 \text{ m}) && (1)\\
&= 5 \times 10^6 \text{ J/year} && (2)\\
&= 5 \times 10^6 \text{ J/year} \left(\frac{1 \text{ year}}{3 \times 10^7 \text{ s}}\right) && (3)\\
&= 0.2 \text{ J/s} && (4)
\end{aligned}$$

This is not much energy, which is why homes don't come with water turbines, even in Oregon.

Because it is such a small value, a factor of two error would not affect our conclusion. Even an order of magnitude or two (e.g. considering a tall apartment complex) wouldn't change our conclusion that adding hydroelectric turbines to buildings is not a great plan. This is the power of a rough order of magnitude estimate: frequently it allows us to answer questions definitively with relatively little effort, which can close off avenues of investigation.