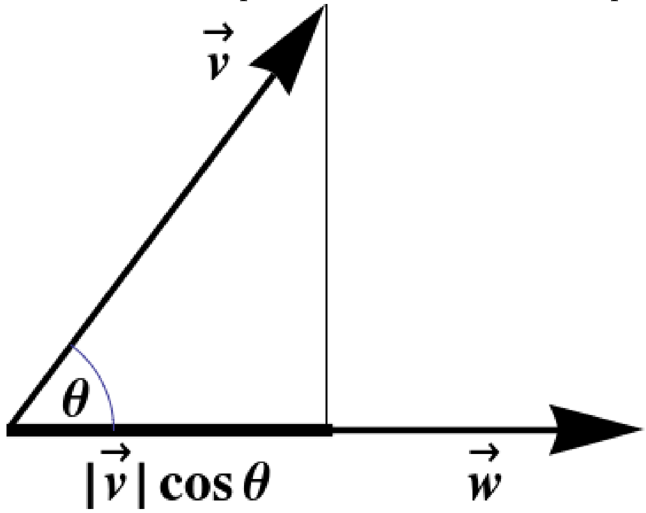
Write one thing you know about the dot product.

Solution The things that you should know about the dot product are:

- It is a mathematical operation that takes two vectors as inputs and outputs a scalar (a number).
- The geometric definition is $\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}|\,\cos\gamma$, where $\vec{v}$ and $\vec{w}$ are the two input vectors and γ is the angle between the vectors.
- The magnitude of a vector can be found by taking the square root of the dot product of the vector with itself $v = |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$.
- The dot product can be used to tell you about the projection of one vector onto another. The value of the dot product of $\vec{v}$ and $\vec{w}$ is the projection of $\vec{v}$ onto $\vec{w}$ times the magnitude of $\vec{w}$.



- The dot product can also be used to find components of vectors. The concept in the previous item simplifies if $\vec{w}$ is a unit vector, in which case the dot product gives the component of $\vec{v}$ in the direction of $\vec{w}$.
- If two vectors are perpendicular, their dot product is zero.
- The dot product is symmetric (i.e. it doesn't matter in what order you input the two vectors).
- The dot product of rectangular basis vectors is

$$\hat{x} \cdot \hat{x} = 1 = \hat{y} \cdot \hat{y} \tag{1}$$
$$\hat{x} \cdot \hat{y} = 0 \tag{2}$$

- The dot product is bilinear, i.e. you can FOIL the dot product as much as you want

$$(a\vec{u} + b\vec{v}) \cdot (c\vec{w} + d\vec{s}) = ac\,\vec{u} \cdot \vec{w} + ad\,\vec{u} \cdot \vec{s} + bd\,\vec{v} \cdot \vec{w} + bd\,\vec{v} \cdot \vec{s} \tag{3}$$

- Once you have found the components of the vectors, you can use the bilinearity and the dot products of the basis vectors to find the algebraic definition of the dot product. (This last item is what most students learn first!)

$$
\begin{aligned}
(v_x\hat{x} + v_y\hat{y}) \cdot (w_x\hat{x} + w_y\hat{y}) &= v_x\, w_x\, \hat{x} \cdot \hat{x} + v_x\, w_y\, \hat{x} \cdot \hat{y} && (4)\\
&+ v_y\, w_x\, \hat{y} \cdot \hat{x} + v_y\, w_y\, \hat{y} \cdot \hat{y} && (5)\\
&= v_x\, w_x\, 1 + v_x\, w_y\, 0 + v_y\, w_x\, 0 + v_y\, w_y\, 1 && (6)\\
&= v_x w_x + v_y w_y && (7)
\end{aligned}
$$

- If the vectors are more than two dimensional, then you just add more terms of the same form to any of the previous expressions involving components. For example, in three dimension $\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z$.