In the past, you have often walked from your home to Checkers. It is 0.5 km SE of your home. You have also often walked to the library. It is 1.0 km N of your home. What is the distance from Checkers to the library. Use **vector** notation and **the dot product** to solve this problem.

Solution If you are trying to find the distance between two objects, ideally you just measure with a tape measure or equivalent. But sometimes you don't have access to the two objects e.g. they are in outer space or tiny under a microscope). In particular, if you are trying to *calculate* the distance in an equation, then you need to do something more. First you must locate the two objects using their positions vectors $\vec{r}$ and $\vec{r'}$ relative to a common origin. Then you must write the position vectors in a common coordinate system $\vec{r} = x\hat{x} + y\hat{y}$, etc. When you subtract the two vectors (use *vector* subtraction!!) you get the vector that goes from one object to the other, $\vec{r} - \vec{r'} = (x - x')\hat{x} + (y - y')\hat{y}$. You can find the length of this (or any other vector) by finding the square root of the dot product of the vector with itself.

$$|\vec{r} - \vec{r''}| = \sqrt{(\vec{r} - \vec{r''}) \cdot (\vec{r} - \vec{r''})} \tag{1}$$

$$= \sqrt{((x - x')\hat{x} + (y - y')\hat{y}) \cdot ((x - x')\hat{x} + (y - y')\hat{y})} \tag{2}$$

$$= \sqrt{(x - x')^2 + (y - y')^2} \tag{3}$$

N.B. Of course, this last expression is the same as the Pythagorean theorem on a triangle whose hypoteneuse is the line segment between the two objects. This is a method of PROVING the Pythagorean theorem. It will help you in future problems to know how to do this calculation the long way, described here.