Electrostatic Potential from Two Charges

- Find a formula for the electrostatic potential $V(\vec{r})$ that is valid everywhere in space for:
  - Two charges $+Q$ and $+Q$ placed on the z-axis at $z' = D$ and $z' = -D$.
  - Two charges $+Q$ and $-Q$ placed on the z-axis at $z' = D$ and $z' = -D$, respectively.

Solution

- Two charges $+Q$ and $+Q$:

$$V(x,y,z) = kQ\left(\frac{1}{\sqrt{x^2+y^2+(z-D)^2}} + \frac{1}{\sqrt{x^2+y^2+(z+D)^2}}\right)$$

- Two charges $+Q$ and $-Q$:

$$V(x,y,z) = kQ\left(\frac{1}{\sqrt{x^2+y^2+(z-D)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+D)^2}}\right)$$

- Simplify your formulas for the special cases of:
  - the x-axis

Solution

* Two charges $+Q$ and $+Q$:

$$V(x,0,0) = \frac{2kQ}{\sqrt{x^2+D^2}}$$

* Two charges $+Q$ and $-Q$:

$$V(x,0,0) = 0$$

  - the z-axis

Solution

* Two charges $+Q$ and $+Q$:

$$\begin{aligned} V(0,0,z) &= kQ\left(\frac{1}{\sqrt{(z-D)^2}} + \frac{1}{\sqrt{(z+D)^2}}\right) && (1) \\ &= kQ\left(\frac{1}{|z-D|} + \frac{1}{|z+D|}\right) && (2) \end{aligned}$$

* Two charges $+Q$ and $-Q$:

$$\begin{aligned} V(0,0,z) &= kQ\left(\frac{1}{\sqrt{(z-D)^2}} - \frac{1}{\sqrt{(z+D)^2}}\right) && (3) \\ &= kQ\left(\frac{1}{|z-D|} - \frac{1}{|z+D|}\right) && (4) \end{aligned}$$

- Discuss the relationship between the symmetries of the physical situations and the symmetries of the functions in these special cases.

Solution

- Two charges $+Q$ and $+Q$: This charge distribution is symmetric across both the x- and z-axes. The function
$$V(x,0,0) = \frac{2kQ}{\sqrt{x^2 + D^2}}$$
is symmetric under the interchange $x \longrightarrow -x$; it is an even function of x. And the function
$$V(0,0,z) = kQ\left(\frac{1}{|z-D|} + \frac{1}{|z+D|}\right) \tag{5}$$
is symmetric under the interchange $z \longrightarrow -z$; it is an even function of z.
- Two charges $+Q$ and $-Q$: This charge distribution is symmetric across the z-axis, but antisymmetric across the x-axis. The function
$$V(x,0,0) = 0$$
is both symmetric and antisymmetric under the interchange $x \longrightarrow -x$; the function 0 is both even and odd. And the function
$$V(0,0,z) = kQ\left(\frac{1}{|z-D|} - \frac{1}{|z+D|}\right) \tag{6}$$
is antisymmetric under the interchange of $z \longrightarrow -z$; it is an odd function of z.