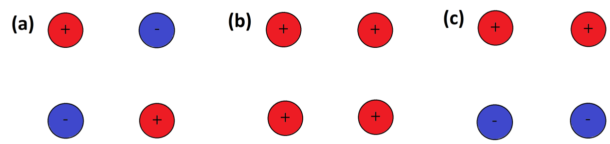
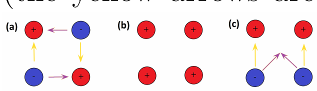
1. Without doing any explicit calculations, use physical reasoning to describe what kind of dependence the dominant term would have for each of these charge orientations, very far away from the charges.



2. Explain physically why the far field potential (V at $s \gg D$) for the Linear Quadrupole (parts a and b) has the dominant term of $\frac{1}{r^3}$, and in particular why it doesn't have $\frac{1}{r}$ or $\frac{1}{r^2}$ dependence?

Solution

1. For the $\frac{1}{r}$ dependence, we know that is the monopole term, so we think of each of these charge orientations as a monopole (i.e. all the charges grouped into one charge at the same position). When we do this, we see that the net charge for (a) and (c) are 0, and so that term is not present for those charge configurations. For (b), we have a net charge of $+4q$, so this one has a non-zero monopole term and the $\frac{1}{r}$ term will dominate at large distances (this orientation will look approximately like a point charge with $Q = +4q$).

   For the $\frac{1}{r^2}$ dependence, we know that is the dipole term, so we analyze these charges as a system of dipoles, all localized at the same point (a valid approximation when we're very far away from the charges!). We can draw the dipole moments (remembering that they point from negative charges to positive charges and that we can choose any coherent set of charge pairs (the yellow arrows are one set, the purple arrows are another, but we can only chose one):
   

   If we localize all these vectors to the same point (i.e. place their tails at the same point), we see that for (a), all the dipole moment vectors would cancel (it doesn't matter if we chose the purple or yellow sets), so that means it has no dipole term and no $\frac{1}{r^2}$ dependence. However, for (c), we can see the dipole components in the horizontal direction would cancel (purple set), but the vertical components would add into something non-zero (both sets), so this configuration has a non-zero dipole term, and since it had no $\frac{1}{r}$ dependence, that means the $\frac{1}{r^2}$ will dominate very far away!

   Now we're left (a), which had 0 for its monopole $\left(\frac{1}{r}\right)$ and dipole $\left(\frac{1}{r^2}\right)$ terms. The $\frac{1}{r^3}$ term is next, and is called the quadrupole term. Following the pattern, we should analyze this as a system of 4 charges localized to a point, but the whole system is just that when we're far away... Well, if we have one quadrupole, there is no way it could be cancelled out like the dipole moments were, so we must have $\frac{1}{r^3}$ dependence that will dominate in the far field.

   Extra Note: You could analyze this term as describing the "quadrupole moment", but mathematically that is a 2nd rank tensor, and goes beyond our ability to draw and the mathematical toolset of this course. However, quadrupole moments have many applications in solid-state and nuclear physics since they can tell you a lot about the underlying symmetry of lattice structures and atomic nuclei just by looking at the external fields. The quadrupole moment also happens to be the signature feature used to detect gravitational waves by LIGO and Nanograv!

2. Like our example (a), the linear quadrupole has net 0 charge, and if we draw the dipole moments, they cancel out as vectors. So that alone is enough to say there is no $\frac{1}{r}$ or $\frac{1}{r^2}$ dependence. We can think of the linear quadruple as a system of 4 charges if we view the $-2Q$ charge at $z = 0$ as two $-Q$ charges stacked on top of each other, and analyzing that as a system of 4 charges, we do in fact have a quadrupole! Which means we should have $\frac{1}{r^3}$ dominate far away from the charges like we found in part b!