Vector Differential: Polar Coordinates:

Find the general form for $d\vec{r}$ in polar coordinates by determining $d\vec{r}$ along the specific paths in the figure below.

- Path 1:

  $$d\vec{r} =$$

- Path 2:

  $$d\vec{r} =$$

If both coordinates are allowed to change simultaneously, by an infinitesimal amount, we could write this $d\vec{r}$ for any path as:

$$d\vec{r} =$$

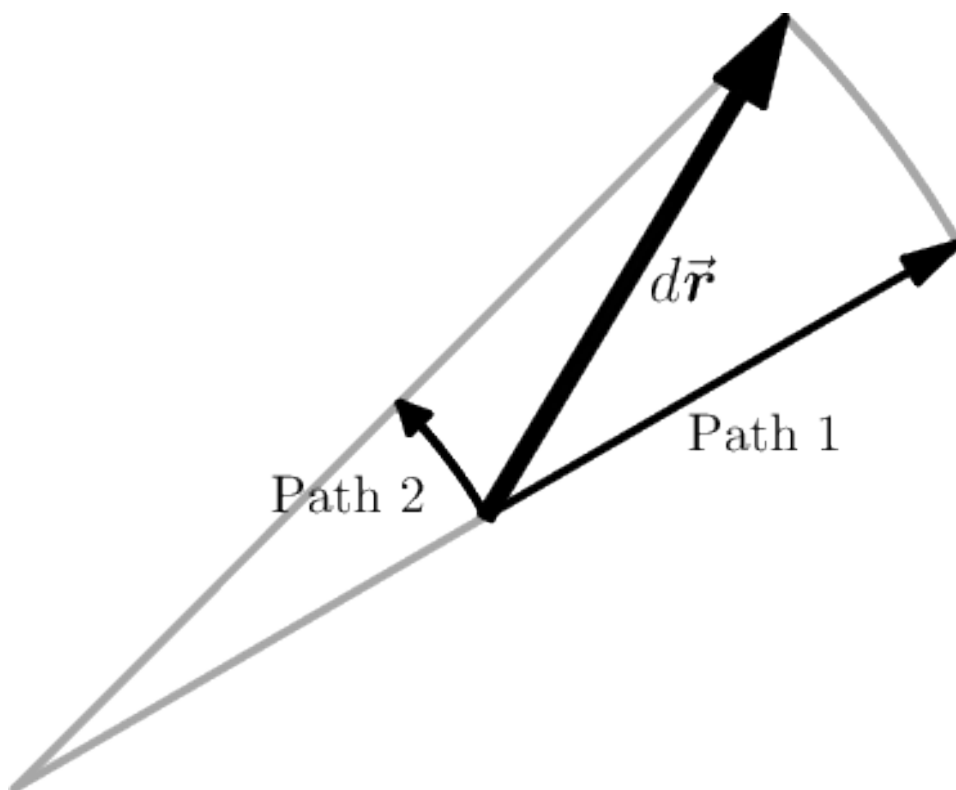This is the general line element in polar coordinates.

Figure 1: Figure 1: $d\vec{r}$ in polar coordinates

Solution

- Path 1:
$$d\vec{r} = dr\,\hat{r}$$

- Path 2:
$$d\vec{r} = rd\phi\,\hat{\phi}$$

- In general:
$$d\vec{r} = dr\,\hat{r} + rd\phi\,\hat{\phi}$$