Find the formulas for the differential surface dA and volume $d\tau$ elements (little chopped pieces of the surface and/or volume) for a plane, for a finite cylinder (including the top and bottom), and for a hemisphere. Make sure to draw an appropriate figure.

Solution You should have obtained the following common surface and volume elements:

In rectangular coordinates:

- $dA = dx\,dy$ for a plane with $z = \text{const}$ in rectangular coordinates. (Cycle x, y, and z in these expressions to find dA on the other three coordinate planes.)
- $d\tau = dx\,dy\,dz$ a small block in rectangular coordinates.

In cylindrical coordinates:

- $dA = s\,ds\,d\phi$ for the top of a cylinder with $z = \text{const}$.
- $dA = s\,ds\,d\phi$ for the bottom of a cylinder with $z = \text{const}$.
- $dA = s\,d\phi\,dz$ for the side of a cylinder with $s = \text{const}$.
- $d\tau = s\,ds\,d\theta$ for a "pineapple chunk" in cylindrical coordinates.

In spherical coordinates:

- $dA = r^2 \sin\theta\,d\theta\,d\phi$ for the surface of a sphere with $r = \text{const}$.
- $dA = r\,dr\,d\phi$ for a plane with $\theta = \pi/2$ on the flat bottom of a hemisphere.
- $d\tau = r^2 \sin\theta\,dr\,d\theta\,d\phi$ for a "pumpkin chunk" in spherical coordinates.