Calculating Total Charge

Each group will be given one of the charge distributions given below: (α and k are constants with dimensions appropriate for the specific example.)

For your group's case, answer the following questions:

1. Find the total charge. (If the total charge is infinite, decide what you should calculate instead to provide a meaningful answer.)

> **Solution** For the spherical cases we end up with finite total charge. However, for the cylindrical cases, we were not told how tall the cylinder was and if our arbitrary height, H, turned out to be infinite, we would have infinite charge. To fix this, we could divide the total change by H (cancelling out our H on the other side and thus our infinity) and give the charge per unit length of the cylinder. This value will be finite and sensable. We should feel confortable declaring arbtrary varibles ofr things we don't know in problems like this because by doing so and going forward with it, we can often make sense of it later in a profound way we wouldn't be able to see easily at the beginning.

2. Find the dimensions of the constants α and k.

> **Solution** We can find this pretty quickly by doing dimensional analysis and know charge density, ρ has dimensions $\frac{[charge]}{[length]^3}$ and any units in our exponents need to end up cancelling out.
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> Sphereical examples:
> 1. $\alpha = [charge]$
> 2. $\alpha = \frac{[charge]}{[length]^3}$
> 3. $\alpha = \frac{[charge]}{[length]}$
>
> Cylindrical examples:
> 1. $\alpha = [charge]$
> 2. $\alpha = \frac{[charge]}{[length]^3}$ and $k = \frac{1}{[length]}$
> 3. $\alpha = \frac{[charge]}{[length]^2}$ and $k = \frac{1}{[length]}$

- **Spherical Symmetry** - A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density:
  a) $\rho(\vec{r}) = \alpha\, r^3$

  > **Solution**
  >
  > $$Q = \iiint \rho(\vec{r}) d\tau = \int_0^{2\pi} \int_0^{\pi} \int_a^b \alpha r^3\, r^2 \sin(\theta) dr d\theta d\phi = 4\pi\alpha \int_a^b r^5 dr = \frac{2}{3}\pi\alpha \left(b^6 - a^6\right)$$

  b) $\rho(\vec{r}) = \alpha\, e^{(kr)^3}$

Solution

$$Q = \iiint \rho(\vec{r})d\tau = \int_0^{2\pi} \int_0^{\pi} \int_a^b \alpha\, e^{(kr)^3}\, r^2 \sin(\theta) dr d\theta d\phi = 4\pi\alpha \int_a^b r^2 e^{(kr)^3} dr = \frac{4\pi\alpha}{3k^3} \left(e^{(kb)^3} - e\right.$$

c) $\rho(\vec{r}) = \alpha\, \frac{1}{r^2}\, e^{(kr)}$

Solution

$$Q = \iiint \rho(\vec{r})d\tau = \int_0^{2\pi} \int_0^{\pi} \int_a^b \frac{\alpha}{r^2}\, e^{kr}\, r^2 \sin(\theta) dr d\theta d\phi = 4\pi\alpha \int_a^b e^{kr} dr = \frac{4\pi\alpha}{k} \left(e^{kb} - e^{ka}\right)$$

- **Cylindrical Symmetry** - A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density:

  a) $\rho(\vec{r}) = \alpha\, e^{(ks)^2}$

  Solution

$$Q = \iiint \rho(\vec{r})d\tau = \int_0^{H} \int_0^{2\pi} \int_a^b \alpha e^{(ks)^2}\, s ds d\phi dz = 2\pi\alpha H \int_a^b s e^{(ks)^2} ds = \frac{\pi\alpha H}{k^2} \left(e^{(kb)^2} - e^{(ka)^2}\right)$$

  b) $\rho(\vec{r}) = \alpha\, \frac{1}{s}\, e^{(ks)}$

  Solution

$$Q = \iiint \rho(\vec{r})d\tau = \int_0^{H} \int_0^{2\pi} \int_a^b \alpha \frac{1}{s} e^{ks}\, s ds d\phi dz = 2\pi\alpha H \int_a^b e^{ks} ds = \frac{2\pi\alpha H}{k} \left(e^{kb} - e^{ka}\right)$$

  c) $\rho(\vec{r}) = \alpha\, s^3$

  Solution For all of these we will need make sense of the height of the cylinder, which we are not told, for now, we will just integrate out to some arbitrary height H:

$$Q = \iiint \rho(\vec{r})d\tau = \int_0^{H} \int_0^{2\pi} \int_a^b \alpha s^2\, s ds d\phi dz = 2\pi\alpha H \int_a^b s^3 ds = \frac{1}{2}\pi\alpha H \left(b^4 - a^4\right)$$