Write one thing you know about the cross product.

Solution The things that you should know about the cross product are:

- It is a mathematical operation that takes two vectors as inputs and outputs a vector.
- The geometric definition is $\vec{u}\times\vec{v} = |\vec{u}|\,|\vec{v}|\,\sin\gamma\,\hat{n}$, where $\vec{u}$ and $\vec{v}$ are the two input vectors, γ is the angle between the vectors, and $\hat{n}$ is a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$.
- A vector perpendicular to both $\vec{u}$ and $\vec{v}$ can be found by taking the cross product $\vec{u}\times\vec{v}$.
- The magnitude of the cross product is the area of the parallelogram whose sides are $\vec{u}$ and $\vec{v}$.
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- The cross product of two parallel vectors is zero.
- The cross product is antisymmetric (i.e. it DOES matter in what order you input the two vectors). If you switch the order of the vectors, the cross product gets an extra minus sign.
- The cross product of rectangular basis vectors is

$$0 = \hat{x}\times\hat{x} = \hat{y}\times\hat{y} = \hat{z}\times\hat{z} \tag{1}$$
$$\hat{x} = \hat{y}\times\hat{z} = -\hat{z}\times\hat{y} \tag{2}$$
$$\hat{y} = \hat{z}\times\hat{x} = -\hat{x}\times\hat{z} \tag{3}$$
$$\hat{z} = \hat{x}\times\hat{y} = -\hat{y}\times\hat{x} \tag{4}$$
$$\tag{5}$$

- The cross product is bilinear, i.e. you can FOIL the cross product as much as you want as long as you don't switch the order of the factors

$$(a\vec{u}+b\vec{v})\times(c\vec{w}+d\vec{s}) = ac\,\vec{u}\times\vec{w} + ad\,\vec{u}\times\vec{s} + bd\,\vec{v}\times\vec{w} + bd\,\vec{v}\times\vec{s} \tag{6}$$

- Once you have found the components of the vectors, you can use the bilinearity and the cross products of the basis vectors to find the algebraic definition of the cross product. (This last item is what most students learn first!) For example:

$$(u_x\hat{x}+u_y\hat{y})\times(v_x\hat{x}+v_y\hat{y}) = u_x\,v_x\,\hat{x}\times\hat{x} + u_x\,v_y\,\hat{x}\times\hat{y} + u_y\,v_x\,\hat{y}\times\hat{x} + u_y\,v_y\,\hat{y}\times\hat{x}\hat{y} \tag{7}$$
$$= u_x\,v_x\,0 + u_x\,v_y\,\hat{z} + u_y\,v_x\,(-\hat{z}) + u_y\,v_y\,0 \tag{8}$$
$$= (u_xv_y - u_yv_x)\,\hat{z} \tag{9}$$

  If the vectors are three dimensional, then you just add more terms of the same form to any of the previous expressions involving components.

- You cannot define the cross product in more than three dimensions (except for the case of 7 dimensions, when you need to use multiplication of the octonions).