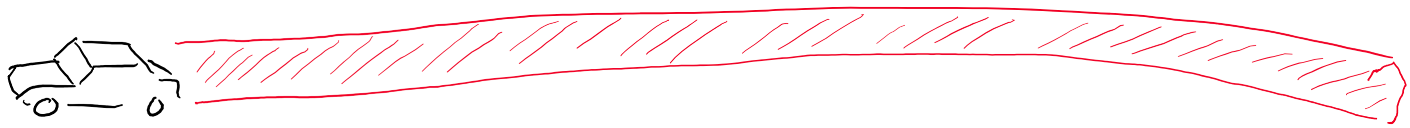
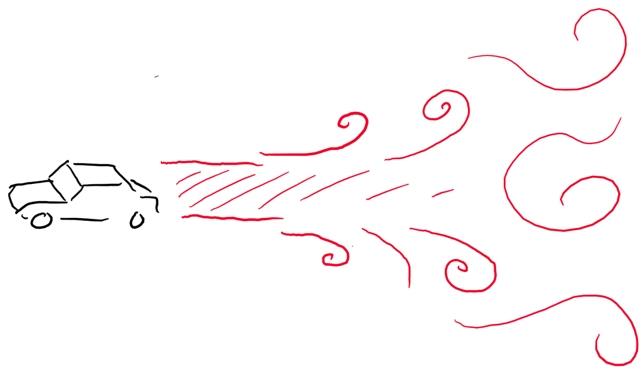
Let's consider where the energy goes when a car is driven down the highway.

One place where the energy ends up is in the wind driven by the car.



The wind right behind the car must be going at the same speed as the car itself. You can imagine that the car is followed by a long "tube" of air that is going at the same speed as the car.

Obviously that is not the case. In reality, the air behind the car has very complicated motion, as the rapidly moving air transfers its energy to the air around it. There is swirling turbulence, vortex shedding, and the details are quite complicated. *Richard Feynman described turbulence as the most important unsolved problem in physics.* Fortunately, since energy is conserved as the flow transitions from a tidy wind trail to a turbulent mess, we can get a reasonable approximation with a coarse-grain model in which we have a clean tube of wind.



1. Estimate the dimensions of a car

> **Solution** Cars are something like a meter or two tall, and a couple of meters wide. The length doesn't really matter for this problem. So the cross-sectional area is a couple of square meters.

2. Estimate the volume of the "wind trail" that a car makes while driving from Corvallis to Portland. (Imagine a clean tube of wind all moving at the same speed as the car.)

> **Solution** The volume of the wind trail will be two square meters times the distance travelled. It takes a couple of hours to get to the Portland airport, so let's call that 120 miles. Since a 5 km run is about the same as a 3 mile run, this makes the distance 200 km or 2×10^5 m. Thus the volume is 4×10^5 m^3.

3. Use the physics concept $K.E. = \frac{1}{2}mv^2$ to find the energy that went into making the wind trail. Assume that $v = 70\frac{\text{miles}}{\text{hour}}$.

Solution At this point, you might need a calculator (or computer). If you look up that 1 mile per hour is 0.45 m/s, then the math isn't too hard.

$$\begin{aligned} v &\approx 70\text{ mph}\frac{0.45\text{ m/s}}{1\text{ mph}} && (1)\\ &= 70\frac{9}{20}\text{ m/s} && (2)\\ &\approx 30\text{ m/s} && (3) \end{aligned}$$

Next, to find the mass of the air, we also need its density. If you look it up, you'll find that the density of air is about 1.3 kg/m^3. So the mass of the air is

$$\begin{aligned} m &\approx 1.3\text{ kg/m}^3 \cdot 4\times 10^5\text{ m}^3 && (4)\\ &\approx 5\times 10^5\text{ kg} && (5) \end{aligned}$$

Putting it all together, we can find that the energy added to the air is

$$\begin{aligned} K.E. &= \frac{1}{2}mv^2 && (6)\\ &\approx \frac{1}{2}\cdot 5\times 10^5\text{ kg}\cdot(30\text{ m/s})^2 && (7)\\ &\approx 2.5\times 10^8\text{ J} && (8) \end{aligned}$$

At this point, we have completed the activity.

Solution

1 Wrap-up discussion

As you might guess, the assumption of a tube of air following the car with a cross-section equal to that of a car isn't quite correct. Engineers put a lot of effort into designing cars to *not* transfer so much energy to the air, i.e. to be aerodynamic. We can account for this with a **drag coefficient**, which is a dimensionless number given by

$$\text{drag coefficient} = \frac{\text{cross-sectional area of wind trail}}{\text{cross-sectional area of car}} \approx 0.3 \qquad (9)$$

So this will reduce our energy by a factor of a third, bringing the energy to $0.3\cdot 2.5\times 10^8\text{ J}\approx 10^8$ J.

1.1 How much gasoline is used?

Some of the gas energy goes into making heat, with something like a 3:1 ratio. Thus, to make 10^8 J of wind energy, I'll need 4×10^8 J of gas energy.

The energy density of gasoline is 4×10^7 J/kg, or 40 MJ/kg, so we need

$$4 \times 10^8 \text{ J} \cdot \frac{1 \text{ kg}}{4 \times 10^7 \text{ J}} \approx 10 \text{ kg of gasoline} \tag{10}$$

Now 1 gallon of gasoline weights 2.8 kg, so we need a bit more than three gallons of gas to drive to Portland. This corresponds to $120/3 = 40$ miles per gallon.

Solution

1.2 Take-home points from car efficiency

$$\text{Total energy of a wind trail} = KE_{\text{trail}} = \frac{1}{2}\rho_{\text{air}} C_d A d v^2 \tag{11}$$

$$\text{Rate energy goes into a wind trail} = \frac{1}{2}\rho_{\text{air}} C_d A d v^3 \tag{12}$$

These relationships can be derived by considering conservation energy *or* by calculating a drag force on the car.

$$|\vec{F}_d| = \frac{1}{2}\rho_{\text{air}} C_d A v^2 \tag{13}$$

Isaac Newton is famous for giving us the tools to analyze physical systems by thinking about forces. **But for many systems you can get even more insight (and solve more easily) by considering conservation of energy!**

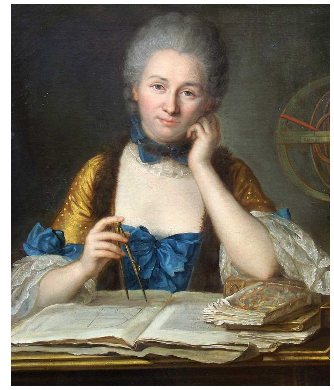
2 Insights to help solve contemporary challenges

Vehicle length KE_{trail} is independent of vehicle length This means long vehicles require no more gas than short ones. Trains and long trucks are more efficient per unit volume transported.

Speed matters A lot. Since the energy lost is proportional to the square of the speed, we could save a lot by driving more slowly. Riding a bicylce is more efficient than driving.

Density of air The loss is proportional to the density of air. Thus proposals like Elon Musk's hyperloop proposal, which puts a train in a low pressure tunnel.

To conclude, this coarse-grained model is a powerful tool for recognizing opportunities and pointing us in productive directions for possible improvements.



Émilie du Châtelet I'd encourage you to read the Wikipedia article on Émilie du Chaâtelet, who first recognized that total energy (with kinetic energy proportional to v^2) is a conserved quantity. She originally developed this experimentally, and her work was built upon by Euler and Lagrange, resulting ultimately in the Lagrangian mechanics taught in Theoretical Mechanics.