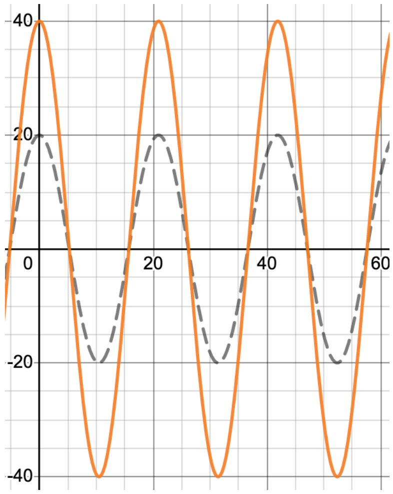
Use the "Massess and Springs" PhET to explore the amplitude and phase parameters of a simple harmonic oscillator.

Attach a 100g block to each spring.

Let the block on the left oscillate with amplitude $20cm$. Then, make it so the right block oscillates with the same frequency ω_0 but the following parameters:

1. Twice the amplitude and relative phase of zero.

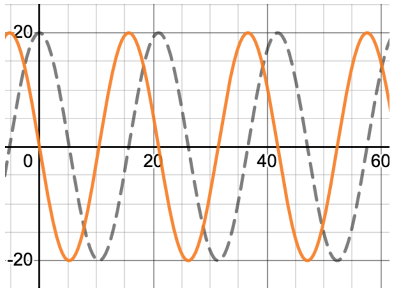
Solution

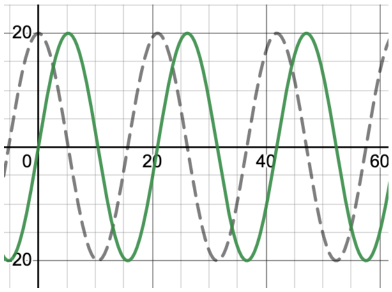


$$
\begin{aligned}
y(t) &= [40\text{ cm}]\cos\omega_0 t \\
&= [20\text{ cm}]e^{i\omega_0 t} + [20\text{ cm}]e^{-i\omega_0 t} \\
&= \text{Re}\left[[40\text{ cm}]e^{i\omega_0 t}\right]
\end{aligned}
$$

2. Same amplitude and relative phase of $+\pi/2$; also $-\pi/2$

Solution

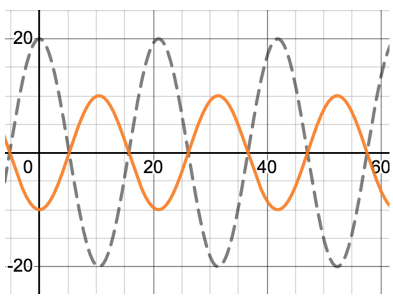




$$\begin{aligned} y(t) &= [20\text{ cm}]\cos(\omega_0 t \pm \tfrac{\pi}{2}) \\ &= \pm[20\text{ cm}]\sin\omega_0 t \\ &= [10\text{ cm}]\cancelto{\pm i}{e^{\pm\frac{i\pi}{2}}} e^{i\omega_0 t} + [10\text{ cm}]\cancelto{\mp i}{e^{\mp\frac{i\pi}{2}}} e^{-i\omega_0 t} \\ &= \text{Re}\left[[20\text{ cm}]e^{\pm\frac{i\pi}{2}} e^{i\omega_0 t}\right] \end{aligned}$$

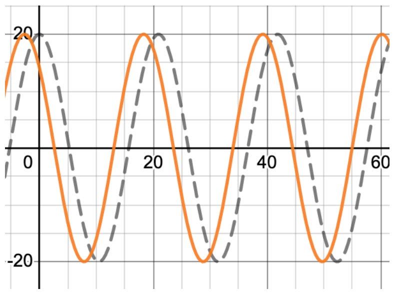
3. Half the amplitude and relative phase $+\pi$; also $-\pi$

Solution



$$\begin{aligned} y(t) &= [10\text{ cm}]\cos(\omega_0 t \pm \pi) \\ &= \mp[10\text{ cm}]\cos\omega_0 t \\ &= [5\text{ cm}]\cancelto{-1}{e^{\pm i\pi}} e^{i\omega_0 t} + [5\text{ cm}]\cancelto{-1}{e^{\mp i\pi}} e^{-i\omega_0 t} \\ &= \text{Re}\left[[10\text{ cm}]\cancelto{-1}{e^{\pm i\pi}} e^{i\omega_0 t}\right] \end{aligned}$$

4. Same amplitude A and phase $+\pi/4$;

Solution 

$$
\begin{aligned}
y(t) &= [20\text{ cm}]\cos(\omega_0 t + \tfrac{\pi}{4}) \\
&= [10\sqrt{2}\text{ cm}]\cos\omega_0 t - [10\sqrt{2}\text{ cm}]\sin\omega_0 t \\
&= [10\text{ cm}]e^{\frac{i\pi}{4}}e^{i\omega_0 t} + [10\text{ cm}]e^{\frac{i\pi}{4}}e^{-i\omega_0 t} \\
&= \mathrm{Re}\left[[20\text{ cm}]e^{\frac{i\pi}{4}}e^{i\omega_0 t}\right]
\end{aligned}
$$

For each situation,

- Plot the position of each block over time. Let the left block be at its maximum displacement at $t = 0$.
- Write an equation that describes the motion of the right block.