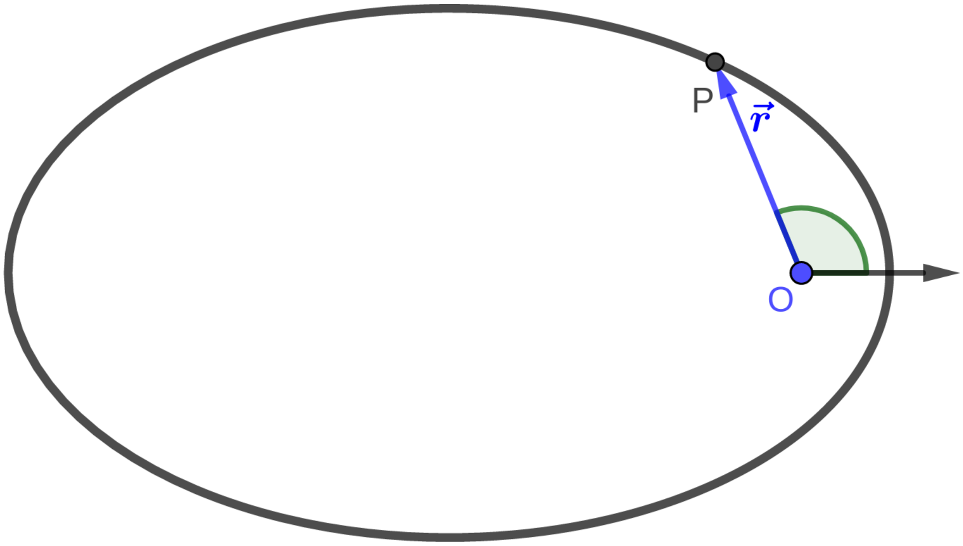
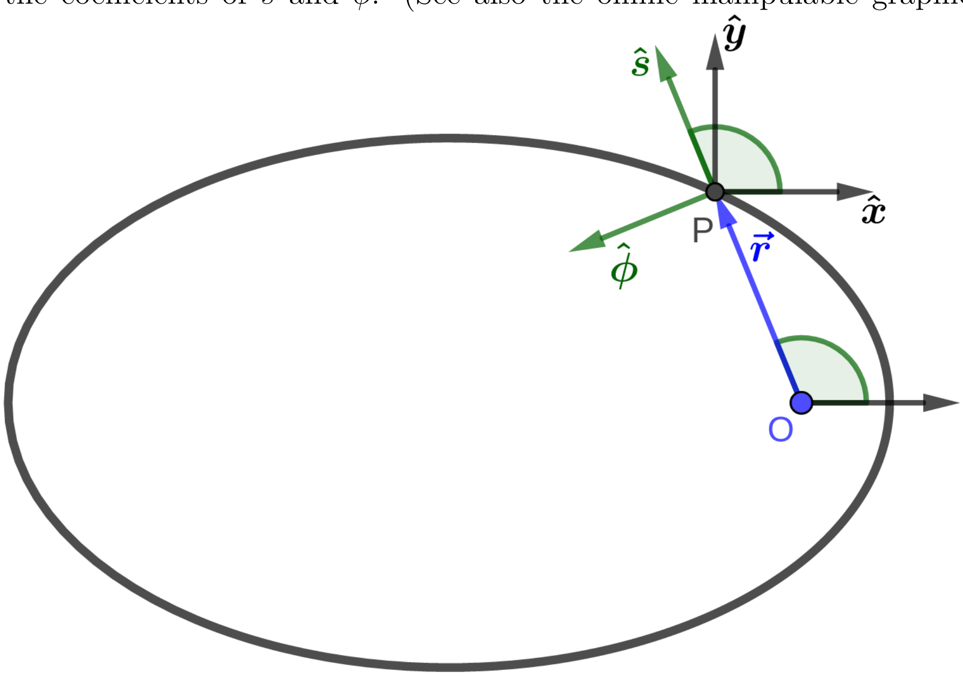
1. On the figure below, draw $\hat{s}$ and $\hat{\phi}$ at P.



2. Find $\frac{d}{dt}\hat{s}$ and $\frac{d}{dt}\hat{\phi}$ in terms of $\hat{s}$ and $\hat{\phi}$.

Solution First use the figure below to write $\hat{s}$ and $\hat{\phi}$ in terms of $\hat{x}$ and $\hat{y}$ which do not change with position or time by drawing the four basis vectors all at point P and reading off the coefficients of $\hat{s}$ and $\hat{\phi}$. (See also the online manipulable graphic version of this figure.)



$$\hat{s} = +\cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Then differentiate, using the fact that $\hat{x}$ and $\hat{y}$ are constant. Reinterpret the answers using the formulas for $\hat{r}$ and $\hat{\phi}$.

$$\begin{aligned}
\dot{\hat{s}} &= -\sin\phi\,\dot{\phi}\,\hat{x} + \cos\phi\,\dot{\phi}\,\hat{y} \\
&= \dot{\phi}\,(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) \\
&= \dot{\phi}\,\hat{\phi} \\
\dot{\hat{\phi}} &= -\cos\phi\,\dot{\phi}\,\hat{x} - \sin\phi\,\dot{\phi}\,\hat{y} \\
&= \dot{\phi}\,(-\cos\phi\,\hat{x} - \sin\phi\,\hat{y}) \\
&= -\dot{\phi}\,\hat{s}
\end{aligned}$$

3. Find $\vec{v}$ in terms of $\hat{s}$ and $\hat{\phi}$.

Solution

$$\begin{aligned}
\vec{v} &= \frac{d\vec{r}}{dt} \\
&= \frac{d}{dt}s\hat{s} \\
&= \frac{ds}{dt}\hat{s} + s\frac{d\hat{s}}{dt} \\
&= \dot{s}\hat{s} + s\dot{\phi}\hat{\phi}
\end{aligned}$$

For more details, please see these lecture notes.