Consider a quantum particle on a ring. At $t = 0$, the particle is in state:

$$|\Phi(t=0)\rangle = \frac{7i}{10}|-2\rangle - \frac{1}{2}|-1\rangle + \frac{1}{2}|0\rangle - \frac{1}{10}|2\rangle$$

1. Find $|\Phi(t)\rangle$

Solution We use the time evolution formula, summing over the rings states given above and pairing each ket with a complex, time-dependent exponential with its corresponding energy:

$$|\Phi(t)\rangle = \sum_{m=-\infty}^{\infty} c_m e^{-i\frac{E_m}{\hbar}t}|m\rangle \; |\Phi(t)\rangle = \frac{7i}{10}e^{-i\frac{E_{-2}}{\hbar}t}|-2\rangle - \frac{1}{2}e^{-i\frac{E_{-1}}{\hbar}t}|-1\rangle + \frac{1}{2}e^{-i\frac{E_0}{\hbar}t}|0\rangle - \frac{1}{10}e^{-i\frac{E_2}{\hbar}t}|2\rangle \; |\Phi(t)\rangle =$$

2. Go to the Ring States GeoGebra applet on the course schedule. Explore changing values of initial coefficents in the applet and see how $Re(\psi)$, $Im(\psi)$, and $|\psi|^2$ change with time.
Then, create the state given above. How do both pieces of the wavefunction and the probability density change with time?

Solution The wavefunction and its probability density both depend on time for the given function. Interesting things to note are that both the real and imaginary parts are time dependent so long as at least one non-zero value for m coefficent in chosen. The probability density also changes in time when we have 2 different values for m that don't have the same magnitude.

3. Calculate the probability that you measure the z-component of the angular momentum to be $-2\hbar$ at time t. Is it time dependent?

Solution $-2\hbar$ correcponds to the ket $|-2\rangle$, so we take the inner product of that state with our overall state and take the norm sqaure of it to get the probability:

$$P(L_z = -2\hbar) = |\langle -2|\Phi(t)\rangle|^2 = \left|\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\langle -2|-2\rangle\right|^2 = \left(-\frac{7i}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right) = \frac{49}{100} \quad (1)$$

This was NOT time dependent!

4. Calculate the probability that you measure the energy to be $\frac{2\hbar^2}{I}$ at time t. Is it time dependent?

Solution $\frac{2\hbar^2}{I}$ correcponds to the ket $|-2\rangle$ AND $|2\rangle$, so we need to add the two probabilities of getting each of these states separately togehter, to get the full probability of measuring this energy:

$$P\left(E = \frac{4\hbar^2}{2I}\right) = |\langle -2|\Phi(t)\rangle|^2 + |\langle -2|\Phi(t)\rangle|^2 = \left|\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right|^2 + \left|\frac{1}{10}e^{-i\frac{2\hbar}{I}t}\right|^2 \quad (2)$$

$$= \left(-\frac{7i}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right) + \left(\frac{1}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{1}{10}e^{-i\frac{2\hbar}{I}t}\right) = \frac{49}{100} + \frac{1}{100} = \frac{1}{2} \quad (3)$$

$$(4)$$

> This was NOT time dependent! Do these results make sense based on what you saw in the applet?