

Unit Learning Outcomes: Integration and Differentiation in Curvilinear Coordinates

In this unit, you will explore integration and differentiation of scalar fields (typically charge and mass density) in 1-, 2-, and 3-dimensions and in rectangular, cylindrical, and curvilinear coordinates. You will also explore vector line integrals in the context of work along non-trivial paths.

What are cylindrical and spherical coordinates? How do you use symmetry to choose an appropriate coordinate system. How are densities in different dimensions defined? How do you use proportional reasoning to describe the functional dependence of a density? How do you use $d\vec{r}$ to describe chopping up curves, surfaces, and volumes? How do you choose limits of integration based on the geometry of a problem? What is the rate of change of a function of more than one variable and how do you calculate it?

Key Activities/Problems

- Charge on a Parabola
- Charge on the Spiral
- Acting Out the Gradient

At the end of this unit, you should be able to:

- Choose a coordinate system that reflects the symmetries of a physical system.
- Write down $d\vec{r}$ from memory in rectangular, cylindrical, and spherical coordinates.
- Use the "use what you know" strategy to evaluate $d\vec{r}$ along a path described in words or figures, even when that path is not along a "coordinate equals constant" curve.
- Integrate a charge or mass density in 1-, 2-, and 3-dimensions to get a total charge.
- Use a vector line integral in rectangular, cylindrical, and spherical coordinates to calculate the work done to move a test charge in an electrostatic field.
- Use the gradient to calculate the rate of change of a multivariable function in any desired direction.