Consider a rod of length L lying on the z-axis. The mass density at $z = 0$ is λ_0 and at $z = L$ is $7\lambda_0$ and the charge density increases linearly.

What is the total charge on the rod?

Solution To find the total charge, I'll chop the rod into tiny segments, find the total charge of each segment, then add up all the charges:

First, I chop the rod into tiny segments with length dz

Then, I find the charge on each segment by multiplying the charge density of the segment by its length:

$$\begin{aligned}\text{tiny charge} &= \lambda(z)\,dz \\ &= \left[\tfrac{6\lambda_0}{L} z + \lambda_0\right]\; dz\end{aligned}$$

Last, I'll add up the charges of all the segments. Since the segments are infinitesimal, the sum is an integral:

$$\begin{aligned}Q &= \int_0^L \lambda(z)\,dz \\ &= \int_0^L \left[\tfrac{6\lambda_0}{L} z + \lambda_0\right]\; dz \\ &= \left[\tfrac{6\lambda_0}{L}\tfrac{z^2}{2} + \lambda_0\, z\right]\Big|_0^L \\ &= 4\lambda_0 L\end{aligned}$$

This is a linear charge density times a length so it has the correct dimensions for charge. If either the density or the length increases, the total charge increases.