The Electrostatic Field Due to a Ring of Charge

1. Use Coulomb's law
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, d\tau'$$
to find the electric field everywhere in space due to a charged ring with radius R and total charge Q.

2. Evaluate your expression for the special case that $\vec{r}$ is on the z-axis.

3. Evaluate your expression for the special case that $\vec{r}$ is on the x-axis.

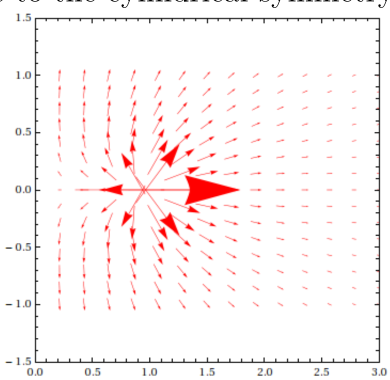
4. Find a series expansion for the electric field at these special locations:

   a) Near the center of the ring, in the plane of the ring;
   b) Near the center of the ring, on the axis perpendicular to the plane of the ring;
   c) Far from the ring on the axis perpendicular to the plane of the ring;
   d) Far from the ring, in the plane of the ring;

Solution The formula for the electric field due to a ring of charge at an **arbitrary** point in space is an eliptic integral, which a computer algebra system such as *Mathematica* is capable of evaluating point by point. The formula is:

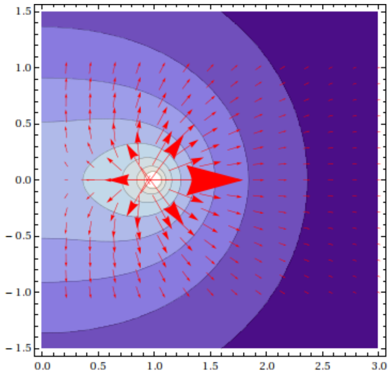
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{2\pi R}\int_0^{2\pi} \frac{[(s\cos\phi - R\cos\phi')\,\hat{x} + (s\sin\phi - R\sin\phi')\,\hat{y} + z\hat{z}]}{\sqrt{s^2+R^2-2Rs\cos(\phi-\phi')+z^2}}\, R\, d\phi'$$

To visualize this solution, it is sufficient to plot the value of the potential in a plane of constant ϕ due to the cylindrical symmetry of the answer.



If we plot the electric field on top of the electrostatic potential, we can see that the first is the (negative) gradient of the second:

$$\vec{E} = -\vec{\nabla} V$$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