Find the formulas for the differential surface and volume elements for a plane, for a finite cylinder (including the top and bottom), and for a sphere. Make sure to draw an appropriate figure.

Solution You should have obtained the following common surface and volume elements:

- $d\vec{A} = \pm\hat{z}\, dx\, dy$ for a plane with $z = $ const in rectangular coordinates.
- $d\vec{A} = \pm\hat{z}\, s\, ds\, d\phi$ for a plane with $z = $ const in polar coordinates.
- $d\vec{A} = +\hat{z}\, s\, ds\, d\phi$ for the top of a cylinder with $z = $ const.
- $d\vec{A} = -\hat{z}\, s\, ds\, d\phi$ for the bottom of a cylinder with $z = $ const.
- $d\vec{A} = +\hat{s}\, s\, d\phi\, dz$ for the side of a cylinder with $s = $ const.
- $d\vec{A} = +\hat{r}\, r^2 \sin\theta\, d\theta\, d\phi$ for the surface of a sphere with $r = $ const.

where we have chosen the sign of the flux, where possible, to correspond to the *outward* pointing normal.

- $d\tau = dx\, dy\, dz$ for a small block in rectangular coordinates.
- $d\tau = r\, dr\, d\phi\, dz$ for a pineapple chunk" in cylindrical coordinates.
- $d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$ for a pumpkin piece" in spherical coordinates.

Note: The formula $d\vec{A} = d\vec{r}_1 \times d\vec{r}_2$ will work for all kinds of complicated surfaces, so we wanted students to get practice in learning how to use it. However, when a surface can be described as a coordinate equals constant" surface in an orthogonal coordinate system, then the cross product is trivial. You can think of the differential area element as an infinitesimal rectangle whose area is just the product of the infinitesimal lengths of the two sides.