

Consider a thin rod of length L lying on the z -axis. Find an algebraic expression for the mass density of the rod if the charge density at $z = 0$ is λ_0 and at $z = L$ is $7\lambda_0$ and you know that the mass density increases linearly.

Solution

$$\lambda(z) = \beta + \alpha z \quad (1)$$

$$\lambda(0) = \beta = \lambda_0 \quad (2)$$

$$\Rightarrow \beta = \lambda_0 \quad (3)$$

$$\lambda(L) = \lambda_0 + \alpha L = 7\lambda_0 \quad (4)$$

$$\Rightarrow \alpha = \frac{6\lambda_0}{L} \quad (5)$$

$$\lambda(z) = \lambda_0 \left(1 + \frac{6}{L} z \right) \quad (6)$$