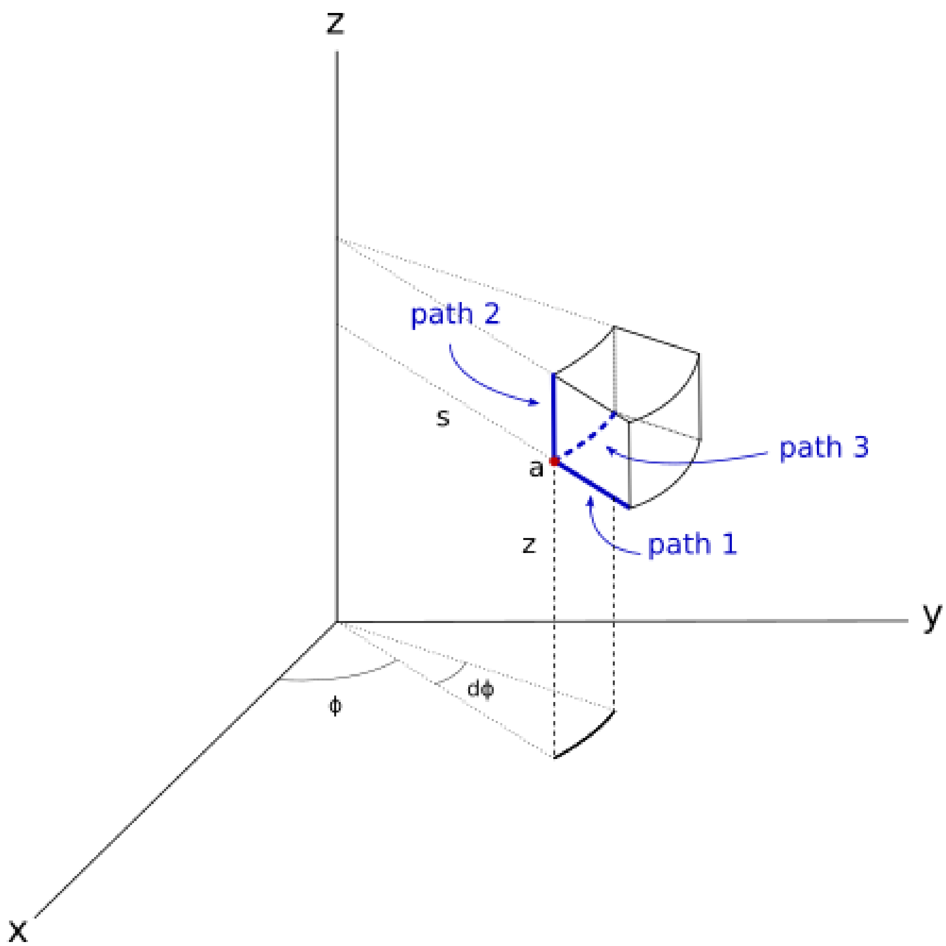
Cylindrical Coordinates Using the first figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate s, ϕ, or z is changing at a time (i.e. along path 1, $dz \neq 0$, but $d\phi = 0$ and $ds = 0$).

Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.
$d\tau =$



Solution

- Path 1:

$$d\ell = ds$$

- Path 2:

$$d\ell = dz$$

- Path 3:

$$d\ell = s\,d\phi$$

$d\ell$ is an infinitesimal length. On path 3, it is necessary to use the arclength formula to turn the infinitesimal angle $d\phi$ into an appropriate length.

$d\tau = (ds)(s\,d\phi)(dz) = s\,ds\,d\phi\,dz$

Spherical Coordinates Using the second figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate r, θ, or ϕ is changing at a time (i.e. along path 1, $d\theta \neq 0$, but $dr = 0$ and $d\phi = 0$).
(Be careful: One path is trickier than the others.)

Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.
$d\tau =$

Solution $d\tau = (dr)(r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,dr\,d\theta\,d\phi$