Eigenvalues and Eigenvectors

Each group will be assigned one of the following matrices.

$$A_1 \doteq \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad A_2 \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad A_3 \doteq \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_4 \doteq \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \qquad A_5 \doteq \begin{pmatrix} 3 & -i \\ i & 3 \end{pmatrix} \qquad A_6 \doteq \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad A_7 \doteq \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$$

$$A_8 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad A_9 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For your matrix:

1. Find the eigenvalues.
2. Find the (unnormalized) eigenvectors.
3. Describe what this transformation does.
4. Normalize your eigenvectors. (Quantum eigenstates are represented by normalized eigenvectors.)

If you finish early, try another matrix with a different structure, *i.e.* real vs. complex entries, diagonal vs. non-diagonal, 2×2 vs. 3×3, with vs. without explicit dimensions.