Time Evolution of a Spin-1/2 System 2

A particles are under the influence of an interaction with a Hamiltonian that is proportional to $\hat{S}_z$:

$$\hat{H} = \omega_0 \hat{S}_z \doteq \begin{bmatrix} \frac{\omega_0 \hbar}{2} & 0 \\ 0 & -\frac{\omega_0 \hbar}{2} \end{bmatrix}$$

Let:
$|\psi(t=0)\rangle = |+\rangle_x$.

1. What is the state particle at a later time t?

2. What is the probability that you would measure $S_x = \frac{\hbar}{2}$ state at time t? Does this probability change with time?

3. What is the probability that you would measure $S_z = \frac{\hbar}{2}$ at time t? Does this probability change with time?

4. What is the probability that you would measure $E = \frac{\omega_0 \hbar}{2}$ at time t? Does this probability change with time?