

Cylindrical Coordinates Using the first figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate s , ϕ , or z is changing at a time (i.e. along path 1, $dz \neq 0$, but $d\phi = 0$ and $ds = 0$).

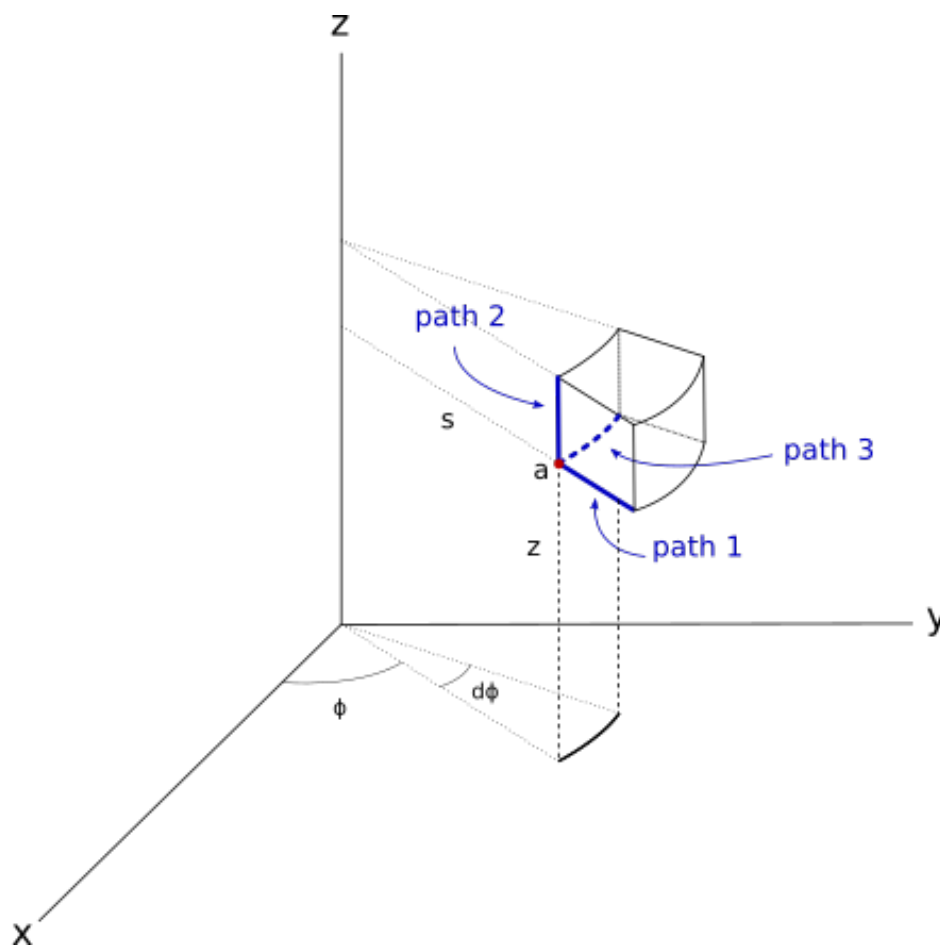
Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.

$d\tau =$



Solution

- Path 1:

$$d\ell = ds$$

- Path 2:

$$d\ell = dz$$

- Path 3:

$$d\ell = s d\phi$$

$d\ell$ is an infinitesimal length. On path 3, it is necessary to use the arclength formula to turn the infinitesimal angle $d\phi$ into an appropriate length.

$$d\tau = (ds)(s d\phi)(dz) = s ds d\phi dz$$

Spherical Coordinates Using the second figure below, determine the **length** $d\ell$ of each of the three paths shown (the three thick lines). Notice that, along any of these three paths, only one coordinate r , θ , or ϕ is changing at a time (i.e. along path 1, $d\theta \neq 0$, but $dr = 0$ and $d\phi = 0$).
(Be careful: One path is trickier than the others.)

Path 1: $d\ell =$

Path 2: $d\ell =$

Path 3: $d\ell =$

Use your results to determine the volume of the region.

$d\tau =$

Solution $d\tau = (dr)(r d\theta)(r \sin \theta d\phi) = r^2 \sin \theta dr d\theta d\phi$

