A solid cylinder with radius R and height H has its base on the x, y-plane and is symmetric around the z-axis. There is a fixed volume charge density on the cylinder $\rho = \alpha z$. If the cylinder is spinning with period T:

1. Find the volume current density.

Solution The volume current density is given by:

$$\vec{J}(\vec{r}) = \rho(\vec{r})\,\vec{v}(\vec{r}) \tag{1}$$

$$= \alpha z\,\frac{2\pi s}{T}\hat{\phi} \tag{2}$$

Notice that the velocity of the different parts of the cylinder is NOT constant, but rather depends on s, the distance from the axis.

2. Find the total current.

Solution This question is a little bit ill-defined. The total current through which surface? It would be logical to pick a surface that maximizes the answer, so we will pick the x, z-plane, where $\phi = 0$. The total current is the flux of the current density through this surface:

$$I_{\text{total}} = \int \vec{J}(\vec{r}) \cdot \hat{n} dA \tag{3}$$

$$= \int_0^R \int_0^H \alpha z\,\frac{2\pi s}{T}\hat{\phi} \cdot ds\,dz\,\hat{\phi} \tag{4}$$

$$= \alpha\frac{2\pi}{T}\int_0^R \int_0^H sz\,ds\,dz \tag{5}$$

$$= \alpha\frac{2\pi}{T}\frac{R^2}{2}\frac{H^2}{2} \tag{6}$$