# 1 State variables for Thermodynamic Systems

In prior courses, when you used partial derivatives, the independent variables were typically space-time coordinates, and orthogonality was an important property. The partial derivatives combine into scalar and vector differential operators: the gradient, divergence, curl, Laplacian, etc. These operators are wonderful and geometric, and totally useless when we leave the comfortable realm of space-time behind, and launch into the seamy underworld of thermodynamics.

The key underlying concept of thermodynamics (and statistical mechanics) is that of *state*. By *state*, we mean a complete macroscopic description of the state of a system. This state uniquely defines the values of all *state variables* (also known as *state functions*, *functions of state* or *state properties*).[1] If you ever find that you can get two different results given the same state, you must conclude that you have not adequately specified the state.

One key property of any system (thermodynamic or otherwise) is that of how many *independent* variables it has. Specifically, this defines how many distinct measurements are required in order to define the state of the system.

## 1.1 PDM

The PDM is a device consisting of a system (usually hidden inside a black box) that can be manipulated by pulling on two strings. The system is designed to prevent the user from changing the angles of the two strings, so that the position of two flags (usually denoted $x_1$ and $x_2$) will define how far the two strings have been pulled. These positions give us *two* state variables. In addition, the tension in each string (denoted $F_1$ and $F_2$) gives us another two state variables, for a total (so far) of four.

These four state variables are not, however, independent. You should have difficulty imagining a scenario in which you change the position of one of the strings without either increasing or decreasing the force on that string! (How would you be pulling it?) With a bit of effort, you can convince yourself that there are only *two* independent variables for this system. It is possible that there is only *one* independent variable, if you cannot change one string's position without also changing the other. The number of independent state variables is thus an experimental question, albeit one that is usually easily answered.

Note that while the number of independent state variables is well defined, the *set* of independent state variables is somewhat more arbitrary. On your PDM, you can choose to set both positions independently, but you could alternatively choose to set both tensions independently. Or you could set one position, and the other tension. Any of these possibilities could be interpreted as "the two independent variables."

we will also note here that it would of course be possible to increase the number of state variables (and independent state variables) by adding another string to the system.

## 1.2 Gas in a piston

Another example of a system with multiple state variables is a simple thermodynamic system. If we define our system to have a given number of particles $N$ (say a gas in a piston), there are several state variables we could either set or measure. These include the volume of the piston, the temperature of

[1] we often try to emphasize my meaning by writing *state variables* but do often refer to them more succinctly as *variables*. we mean the same thing by this.

the gas, the pressure of the gas, and its entropy. These are not all independent, because if you wanted to change the volume (say without changing the temperature), you would have to push on the piston. As it turns out, if we keep the number $N$ fixed, the number of independent state variables is two.

## 1.3 Magnetization of iron

Another example from thermodynamics is that of the magnetization of a ferromagnet (say, a chunk of iron). As in any thermal system, the temperature and entropy are state variables, but in this case the external magnetic field and the magnetization are also state variables. As in the previous case, we have two independent state variables.

The set of state variables is flexible, and depends how we choose to define our system. In the previous example of a piston, we ignored the magnetic field, either because we thought it was irrelevent, or chose to define our system as one with negligibly small magnetic field. Similarly, in the case of iron, we chose to ignore the volume of the iron and the pressure, because we deemed them irrelevant to the question of magnetization (as long as your pressure is not extreme). This flexibility is common to most problems you have encountered in physics, but you may not have seen it play such a large role.

## 1.4 Circuits

Another system we could consider would be a circuit. If you were given a black box (or a chip) with $N$ leads on it, you could consider it to be a system, which you could manipulate by applying voltage to each of those leads, and you could measure the current through each connecting wire, giving you $2N$ state variables, of which probably only $N$ are independent (since you can imagine connecting each wire to a different ideal battery, and measuring all the currents).

This example highlights a question you must ask yourself whenever defining *state* in a system, which is whether your set of independent variables uniquely define the *state* of the system. It is entirely possible (and perhaps even likely, in the case of an integrated circuit) that your system will have some internal state, which could mean that the values of your state variables do not uniquely define the state of your system. This happens when your system has "memory" or displays *hysteresis*. If this is the case, you must use caution when applying the mathematical methods we will be discussing here.