You are given the following equations:

$$pV = Nk_BT \tag{1}$$

$$U = \frac{3}{2}Nk_BT \tag{2}$$

$$S = Nk_B\left\{\ln\left[\frac{V}{N}\left(\frac{mU}{3\pi N\hbar^2}\right)^{3/2}\right] + \frac{5}{2}\right\} \tag{3}$$

The isothermal and isentropic (adiabatic) bulk moduli, β_T and β_S respectively, are defined as follows:

$$\beta_T = -V\left(\frac{\partial p}{\partial V}\right)_T \qquad \beta_S = -V\left(\frac{\partial p}{\partial V}\right)_S$$

1. Find an expression for β_T in terms of p, V, and T (at the end, simplify to as few variables as possible)

Solution To find β_T you must evaluate a straightforward partial derivative using the first equation, the ideal gas law.
Divide the left equation ($pV = Nk_BT$) by V to isolate p;

$$p = \frac{Nk_BT}{V} \tag{4}$$

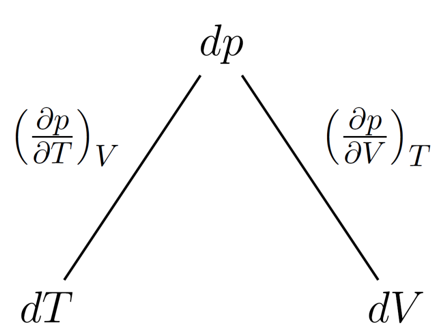
Evaluate the partial derivative;

$$\left(\frac{\partial p}{\partial V}\right)_T = -\frac{Nk_BT}{V^2} \tag{5}$$

Thus,

$$\beta_T = -V\left(\frac{\partial p}{\partial V}\right)_T = \frac{Nk_BT}{V} \tag{6}$$

Note: Alternatively, one can zap the given equation with d, isolate dp, and identify the derivative by comparing the result to the total differential of $p(V, T)$; $dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT$. The total differential can also be represented as a chain rule diagram element (as can any total differential);



2. Find an expression for β_S in terms of p, V, T and S (at the end, simplify to as few variables as possible).

Solution See Ethan's class notes (OneNote file) for a detailed solution.