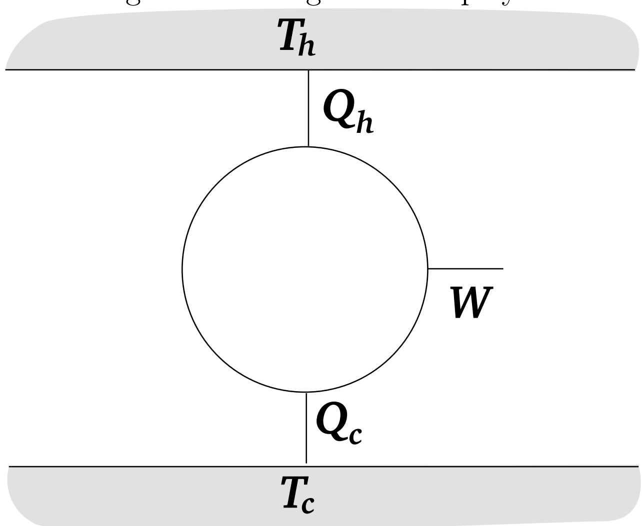
A **heat engine** is a device that accepts energy in the form of heat from something hot, and uses that energy to do work. The *Kelvin Formulation* of the Second Law states that this cannot be *all* that a heat engine does. In fact, a heat engine will also "waste" energy by heating something cool. Thus the energy you put in from the "hot place" will not all get used to do useful work.

I will diagram heat engines as displayed in this picture. The heat engine contains several parts.



- At the top and bottom are hot and cold **heat sinks**. A **heat sink** is an object that is big enough (has a large enough heat capacity) to provide energy by heating without changing its temperature very much. The operator of the engine will have to keep these two heat sinks at fixed temperature, which means burning fuel to warm up the hot sink, and using something like a radiator to keep the cold sink cool by letting it dissipate to the outside world.

- In the middle of the picture is the engine itself, which will contain some sort of a **working substance** that is (most likely) alternately heated and cooled. A key idea is that the engine itself must be cyclic, meaning that it will return to precisely its original state.

- There is some amount of heat Q_h transfered from the hot sink to the engine, and some other amount of heat Q_c (both taken to be positive numbers) transfered from the engine to the cold sink. By the First Law, the difference between these must be the work.

$$W = Q_h - Q_c$$

It may seem like heat engines (and steam engines) are a bit old-fashioned, but about 80% (according to wikipedia) of electric power in the world is generated by steam turbines which are simple heat engines. So, the appliations are not stuck in the 1800s, although heat engines were pretty well understood in the 1800s. More modern systems like thermoelectrics also operate as heat engines. One can even view a photovoltaic solar cell as a poor heat engine, operating between the hot sun and the cool earth. In both thermoelectrics and solar cells, the efficiency is very poor. There is a large gap in efficiency between the fundamental limit that we derive below, and what has been acheived to date. This gap inspires many scientists to continue improving the design of thermoelectrics and photovoltaics.

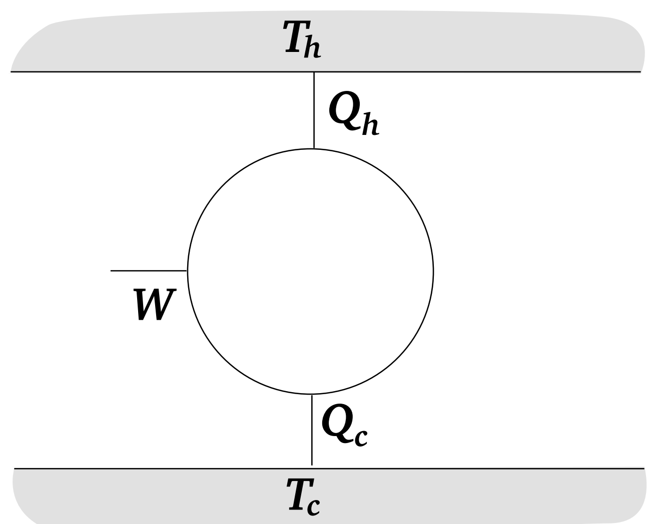
1 Efficiency

The **efficiency** in general is what you get out divided by what you put in. In this case, what we have to put into a heat engine is the energy added to keep the hot reservoir hot Q_h, and what we get out is the net work W so

$$\eta = \frac{W}{Q_h} \tag{1}$$
$$= \frac{Q_h - Q_c}{Q_h} \tag{2}$$
$$= 1 - \frac{Q_c}{Q_h} \tag{3}$$

So clearly, we'd like to minimize the amount of heat sent to the cold sink. This also has environmental advantages, e.g. to avoid warming a river.

If each step is done *reversibly*, then we could run a heat engine in reverse, and have a refrigerator. Thus we *do* work on the fridge, and cool off the cold sink while warming up the hot sink.



The efficiency of any reversible heat engine must be the same as the efficiency of any reversible refrigerator. We can see this by using a heat engine to drive a refrigerator. By choice the work done by the engine is the same as the work done on the fridge. If the efficiencies of the fridge and engine differ, then there will be a net transfer of heat either from hot sink to cold sink or from cold sink to hot sink. The former would be more reasonable and natural, but the latter would be crazy (i.e. it violates the *Clausius formulation* of the Second Law), which means that the fridge cannot be *more efficient* than the engine. However, if both fridge and engine are reversible, then if the fridge is *less efficient* than the engine, then we could run the thing in reverse, and get the crazy situation happening, in which nothing changes except that heat is transferred from a cold place to a hot place... and that just isn't natural!

So we can only conclude that every possible reversible heat engine operating between the same two temperatures must have the same efficiency!

For a reversible engine, the net entropy change of system plus surroundings must be zero over each cycle. Since the entropy change of the system itself for a cyclic process is always zero, even for an irreversible engine, we only need consider the entropy change of the surroundings.

2 Efficiency of a reversible heat engine

To find the efficiency of a reversible heat engine, we just need to recognize that the change in entropy of the system plus surroundings must be equal to zero. Since the heat engine itself must return to its original state after each cycle, its entropy cannot change after a full cycle. So we just need to ensure that any entropy *lost* by the hot bath must be equal to the entropy *gained* by the cool bath.

In your small groups, use the assumption that the entropy lost by the hot bath is equal to the gain in entropy of the cold bath to solve for the efficiency of a heat engine.

Solution The entropy lost by the hot bath is

$$|\Delta S_H| = Q_H/T_H \tag{4}$$

since its temperature remains constant. By the same reasoning, the entropy gained by the cool bath will be

$$|\Delta S_C| = Q_C/T_C \tag{5}$$

Setting these two equal tells us that

$$\begin{aligned} Q_H/T_H &= Q_C/T_C \quad (6) \\ \frac{Q_H}{Q_C} &= \frac{T_H}{T_C} \quad (7) \\ \eta &= 1 - \frac{Q_C}{Q_H} \quad (8) \\ &= 1 - \frac{T_C}{T_H} \quad (9) \end{aligned}$$

This is the famous **Carnot efficiency**. Any reversible heat engine must have this efficiency.