Consider a quantum particle on a ring. At $t = 0$, the particle is in state:

$$|\Phi(t=0)\rangle = \frac{7i}{10}|-2\rangle - \frac{1}{2}|-1\rangle + \frac{1}{2}|0\rangle - \frac{1}{10}|2\rangle$$

1. Find $|\Phi(t)\rangle$

Solution We use the time evolution formula, summing over the rings states given above and pairing each ket with a complex, time-dependent exponential with its corresponding energy:

$$|\Phi(t)\rangle = \sum_{m=-\infty}^{\infty} c_m e^{-i\frac{E_m}{\hbar}t}|m\rangle \; |\Phi(t)\rangle = \frac{7i}{10}e^{-i\frac{E_{-2}}{\hbar}t}|-2\rangle - \frac{1}{2}e^{-i\frac{E_{-1}}{\hbar}t}|-1\rangle + \frac{1}{2}e^{-i\frac{E_0}{\hbar}t}|0\rangle - \frac{1}{10}e^{-i\frac{E_2}{\hbar}t}|2\rangle \; |\Phi(t)\rangle =$$

2. Calculate the probability that you measure the z-component of the angular momentum to be $-2\hbar$ at time t. Is it time dependent?

Solution $-2\hbar$ correcponds to the ket $|-2\rangle$, so we take the inner product of that state with our overall state and take the norm sqaure of it to get the probability:

$$P(L_z = -2\hbar) = |\langle -2|\Phi(t)\rangle|^2 = \left|\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\langle -2|-2\rangle\right|^2 = \left(-\frac{7i}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right) = \frac{49}{100} \quad (1)$$

This was NOT time dependent!

3. Calculate the probability that you measure the energy to be $\frac{2\hbar^2}{I}$ at time t. Is it time dependent?

Solution $\frac{2\hbar^2}{I}$ correcponds to the ket $|-2\rangle$ AND $|2\rangle$, so we need to add the two probabilities of getting each of these states separately togehter, to get the full probability of measuring this energy:

$$P\left(E = \frac{4\hbar^2}{2I}\right) = |\langle -2|\Phi(t)\rangle|^2 + |\langle -2|\Phi(t)\rangle|^2 = \left|\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right|^2 + \left|\frac{1}{10}e^{-i\frac{2\hbar}{I}t}\right|^2 \quad (2)$$

$$= \left(-\frac{7i}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{7i}{10}e^{-i\frac{2\hbar}{I}t}\right) + \left(\frac{1}{10}e^{i\frac{2\hbar}{I}t}\right)\left(\frac{1}{10}e^{-i\frac{2\hbar}{I}t}\right) = \frac{49}{100} + \frac{1}{100} = \frac{1}{2} \quad (3)$$

$$(4)$$

This was NOT time dependent! Do these results make sense based on what you saw in the applet?